

# **BOÄ MOÂN TOAÙN ÖÜNG DUÏNG - ÑHBK**

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## **BAØI 2: HAØM SOÁ**

# NOÀI DUNG

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1- ÑÒNH NGHÓA

~~HAØM SOÁ~~  
2- HAØM SOÁ

~~NGÖÖÏC~~  
3- HAØM LÖÖÏNG GIAÙC

~~NGÖÖÏC~~  
4- HAØM

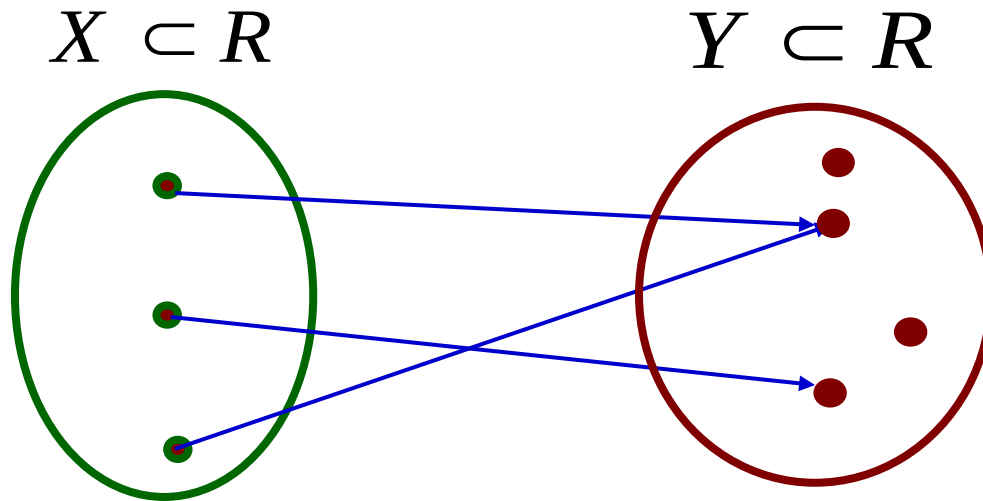
HYPERBOLIC

# ÑÒNH NGHÓA HÀØM SỐÁ

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**Hàm số  $f: X \subset \mathbb{R} \rightarrow Y \subset \mathbb{R}$  là**

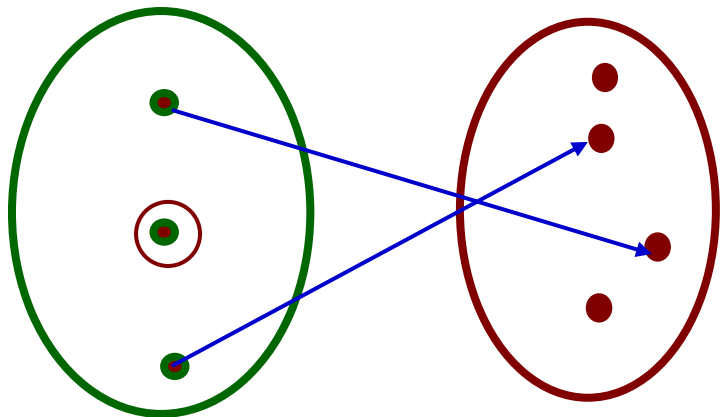
**Quy luật tương ứng mỗi  $x \in X$  với duy nhất  $y = f(x) \in Y$ .**



**$x$  : biến;  $y = f(x)$  : ảnh của  $x$   
qua ánh xạ  $f$**

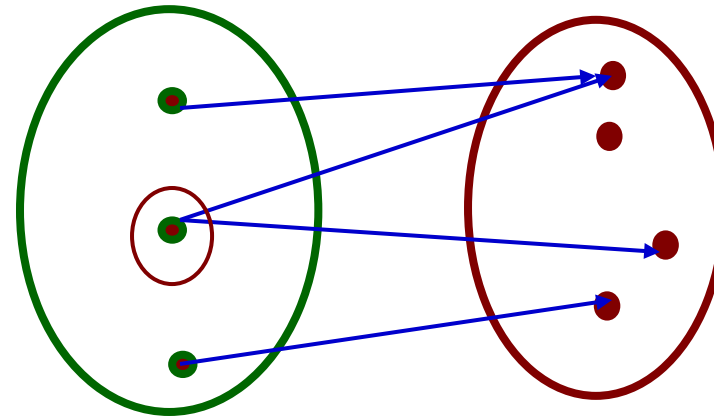
$X \subset R$

$Y \subset R$



$X \subset R$

$Y \subset R$



• Không là ảnh xạ vì  
 có 1 biến  $x$  không  
 có ảnh.

• Không là ảnh xạ vì  
 có 1 biến  $x$  có 2  
 ảnh.

Miền xác định:  $D_f = \{x / f(x)$   
 có nghĩa}

Miền giá trị:  $\{y \in R / \exists x \in D_f, y = f(x)\}$

# XÀÙC ÑÒNH HÀØM SỐÁ QUA BIEÁU THÒÙC

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**Bieáu  
thòùc:**

**Quen thuoác (daing**

**VD:  $y = x^2, y = e^x$**

**Daing**  $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$

**tham soá**

**VD:  $x = 1 + t, y = 1 - t \rightarrow$**

**VD:  $x = a \cos t, y = a \sin t \rightarrow$**

**Ñöông tron**

**Daing aân  $F(x, y) = 0 \Rightarrow y = f(x)$**

**(implicit)**  
**VD: Ntron  $x^2 + y^2 - \frac{x^2}{16} + \frac{y^2}{9} - 1 = 0$**

**$4 = 0,$**

# Hàm $y = x^\alpha$

❖ **MXÑ** :  $\alpha$  **tõ** nhieân  $\Rightarrow D=R$ ,  $\alpha$  **nguyeân âm**  $\Rightarrow D=R \setminus \{0\}$ ,

$\alpha \in R$  (noùi chung)  $\Rightarrow D=(0, +\infty)$  (hàm caên: tuyø tính chaün leû)

❖ **Tính ñôn ñieâu** (chæ xeùt  $x > 0$ ):  $\alpha > 0 \rightarrow$   
 $\lim_{x \rightarrow +\infty} x^\alpha = 0 (\alpha < 0)$   $\lim_{x \rightarrow +\infty} x^\alpha = +\infty (\alpha > 0)$

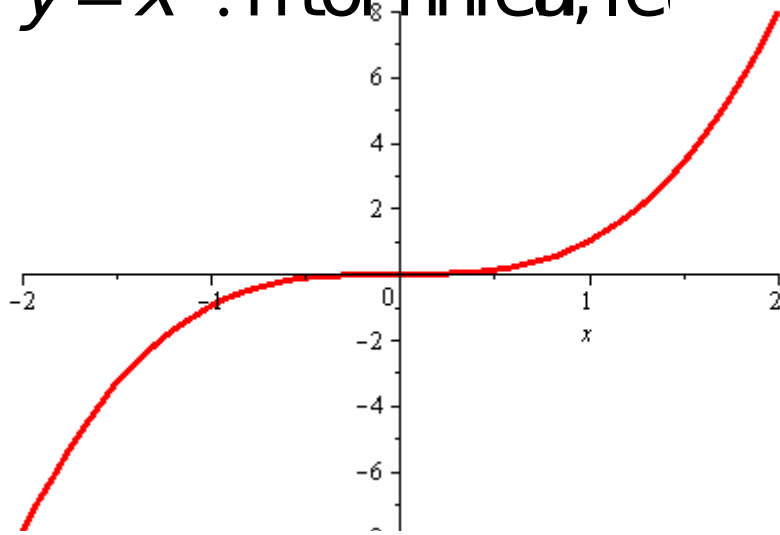
**Taêng**,  $\alpha < 0 \rightarrow$  **Giaûm**

❖ **Giòu haïn**

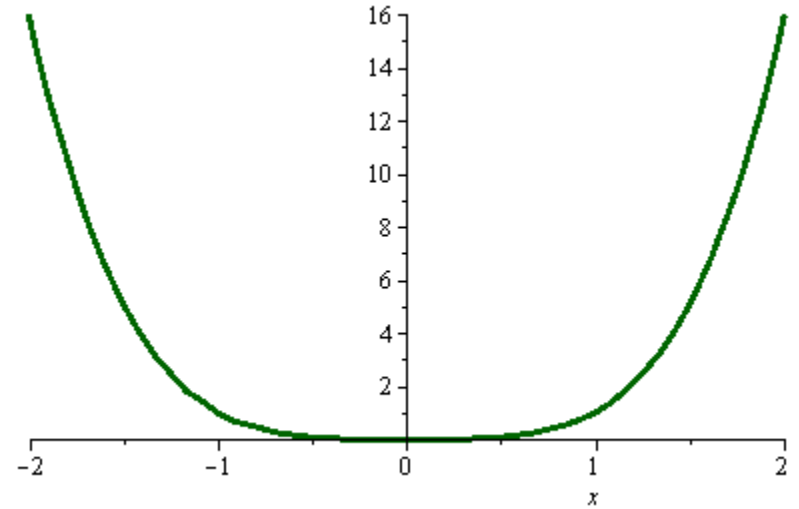
# ÑÒÀ THÒ HAØM LUYÕ THÖØA

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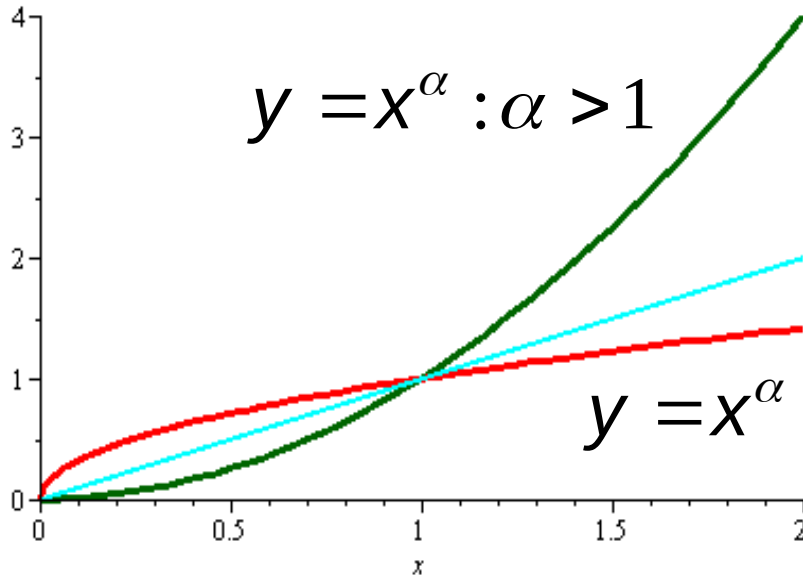
$$y = x^n : n \text{ tõi nhieâ, le}$$



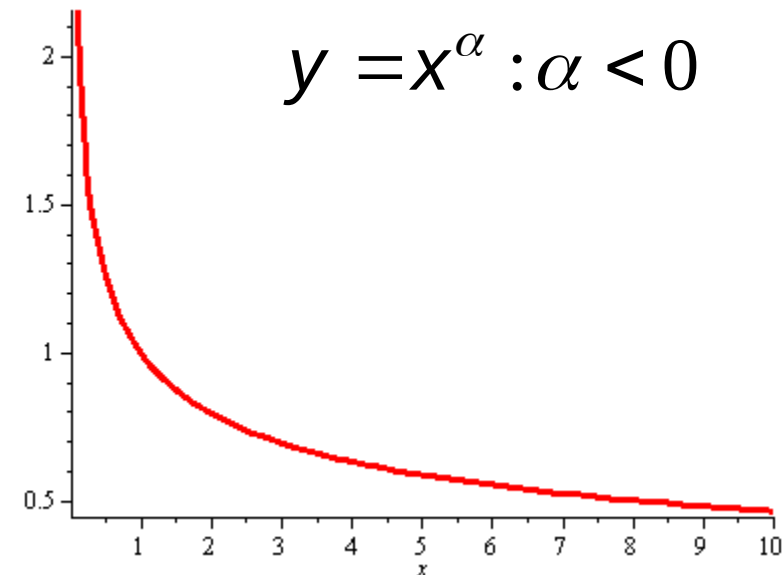
$$y = x^n : n \text{ tõi nhieâ, chæ}$$



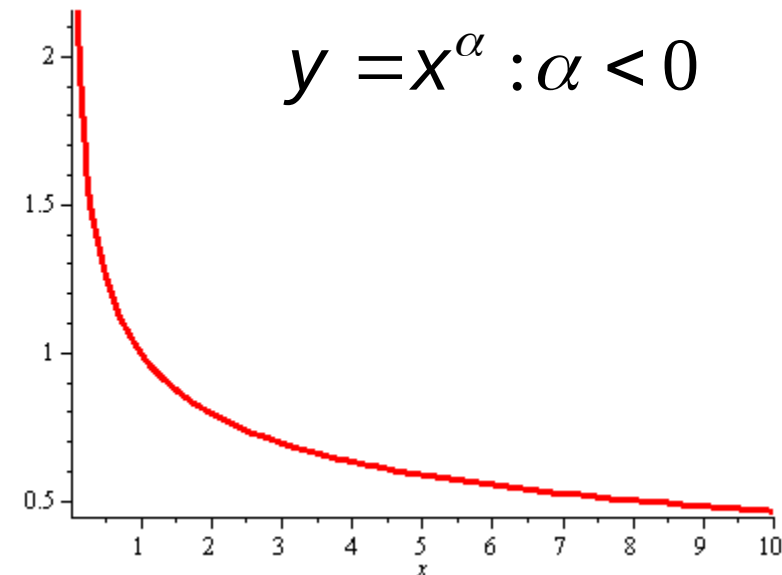
$$y = x^\alpha : \alpha > 1$$



$$y = x^\alpha : 0 < \alpha < 1$$



$$y = x^\alpha : \alpha < 0$$



# HAØM MUÕ

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$$y = a^x \quad (a > 0)$$

❖ **MXÑ: R; MGT: (0,**

**†<sup>∞</sup>) Nôn ñieäu : a > 1 ⇒ Haøm taêng, 0 < a < 1 ⇒  
Haøm giaûm**

$$a > 1: \lim_{x \rightarrow +\infty} a^x = +\infty \text{ \& } \lim_{x \rightarrow -\infty} a^x = 0;$$

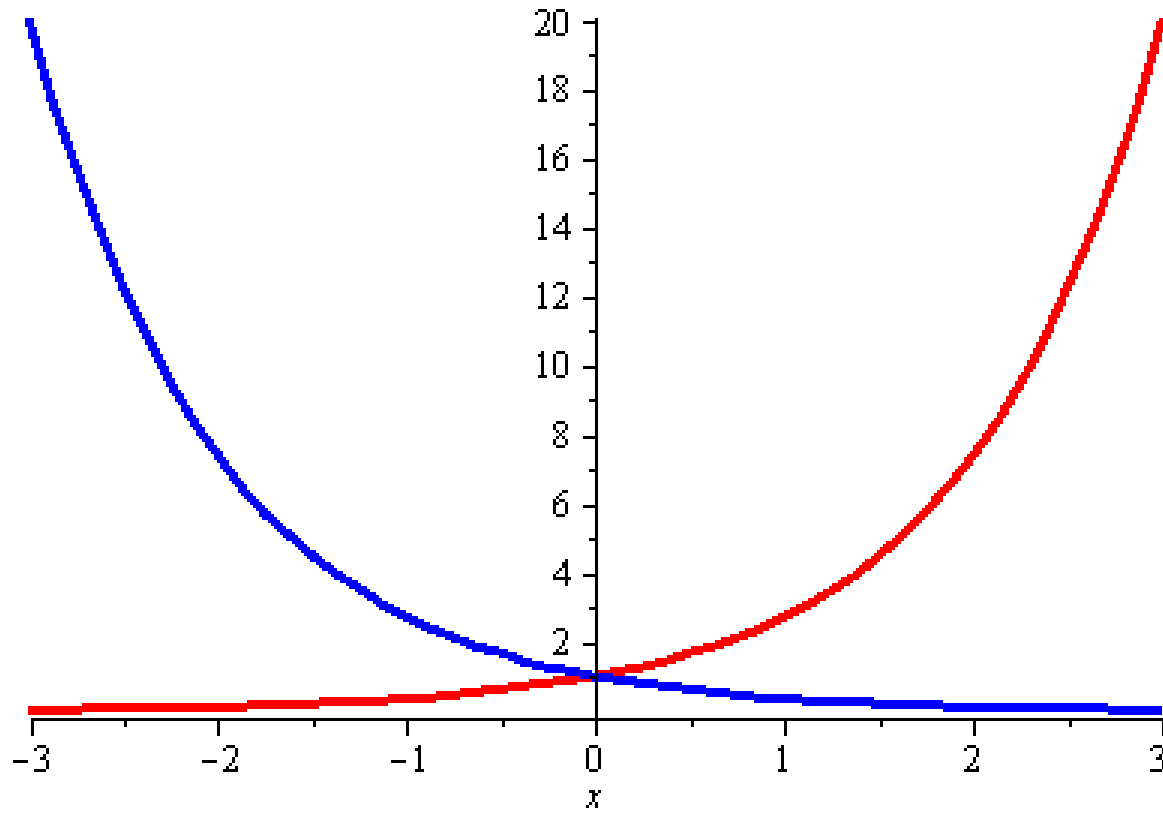
❖ **Giôùi haïn :**

$$0 < a < 1: \lim_{x \rightarrow +\infty} a^x = 0 \text{ \& } \lim_{x \rightarrow -\infty} a^x = +\infty$$

## ĐỒ THỊ HÀM MŨ

$$y = a^x, 0 < a < 1$$

$$y = a^x, a > 1$$



# HAØM logarit

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$$y = \log_a x \quad (a > 0)$$

❖ **MXÑ:  $x > 0$ , MGT :**

**R**

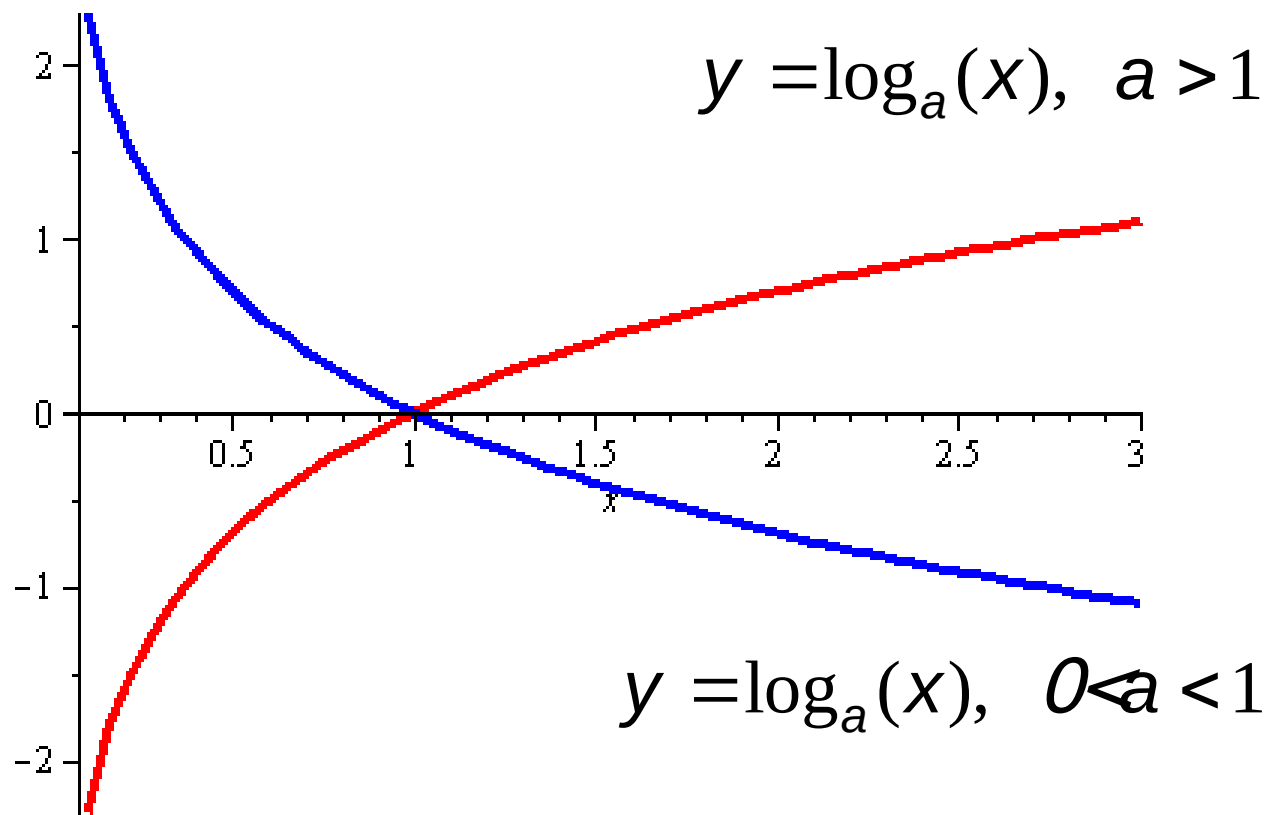
❖ **Ñôn ñieäu:  $a > 1 \Rightarrow$  TAÊNG ,  $0 < a < 1 \Rightarrow$  GIAÛM**

❖

**Ghain**

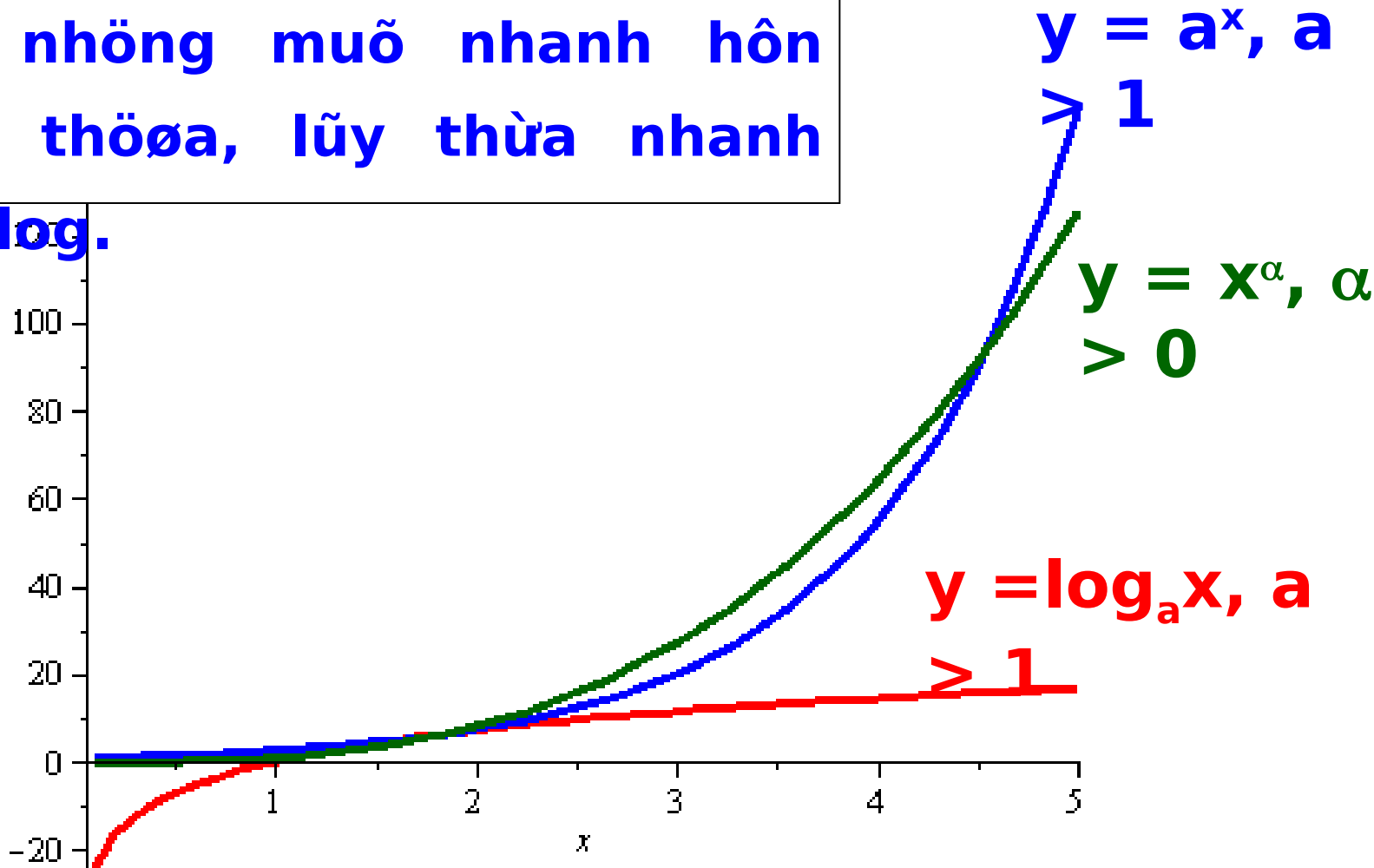
$$\left\{ \begin{array}{l} a > 1: \lim_{x \rightarrow +\infty} \log_a x = +\infty \text{ \& } \lim_{x \rightarrow 0^+} \log_a x = -\infty \\ 0 < a < 1: \lim_{x \rightarrow +\infty} \log_a x = -\infty \text{ \& } \lim_{x \rightarrow 0^+} \log_a x = +\infty \end{array} \right.$$

# ĐỒ THỊ HÀM LOGARIT



# HAØM MUỖ, LOGARIT: SO SAØNH VỚI LUYÕ THỪA khi $x \rightarrow +\infty$

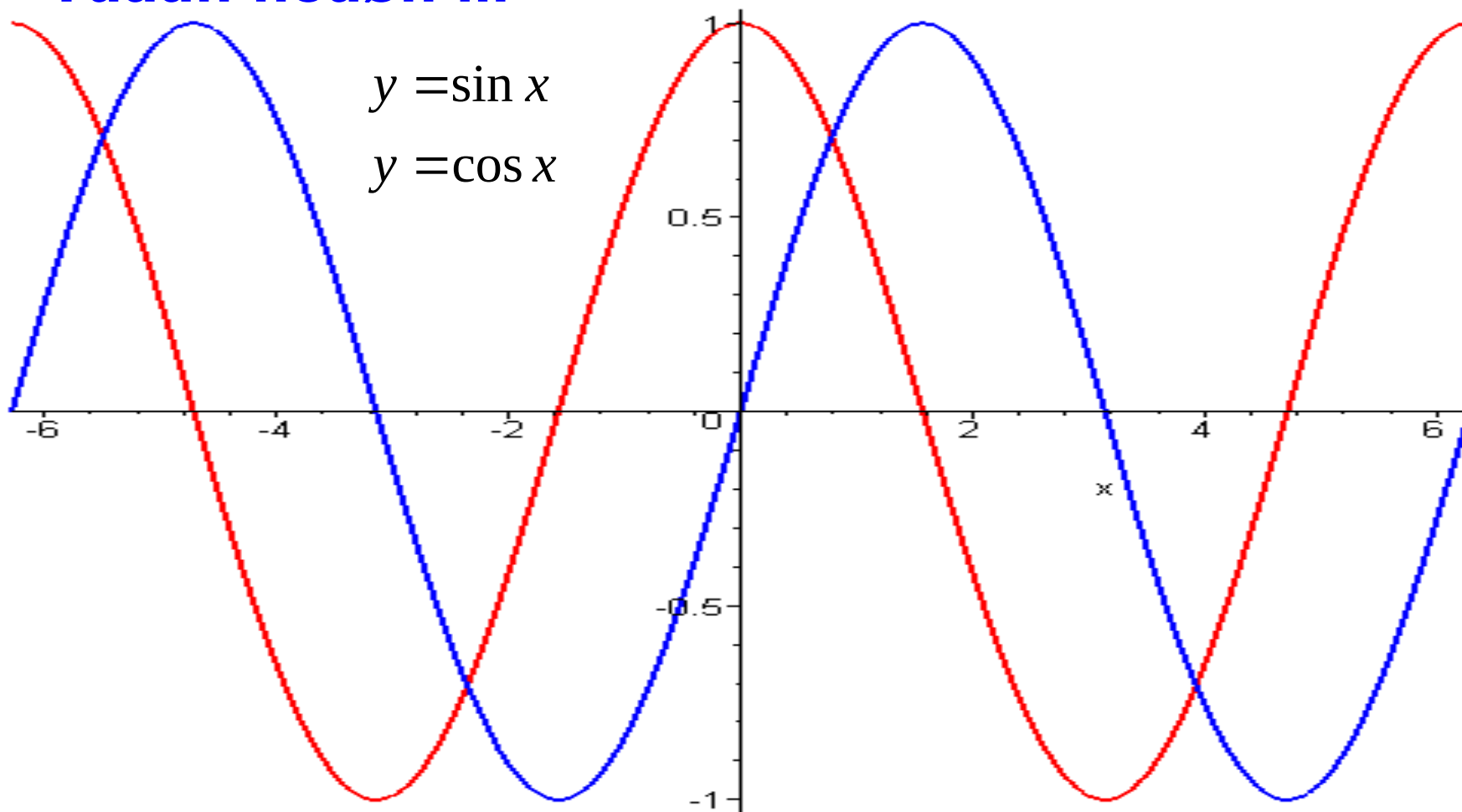
Khi  $a > 1$  &  $\alpha > 0$ : Cường  $\uparrow$ ,  $\rightarrow +\infty$ , những mũ nhanh hơn lũy thừa, lũy thừa nhanh hơn log.



# HAØM LÖÖING GIAÙC: $\sin x$ , $\cos x$

$y = \sin x$ ,  $y = \cos x \Rightarrow$  MXÑ:  $\mathbb{R}$ , MGT:  $[-1, 1]$ ,

Tuần hoàn ...



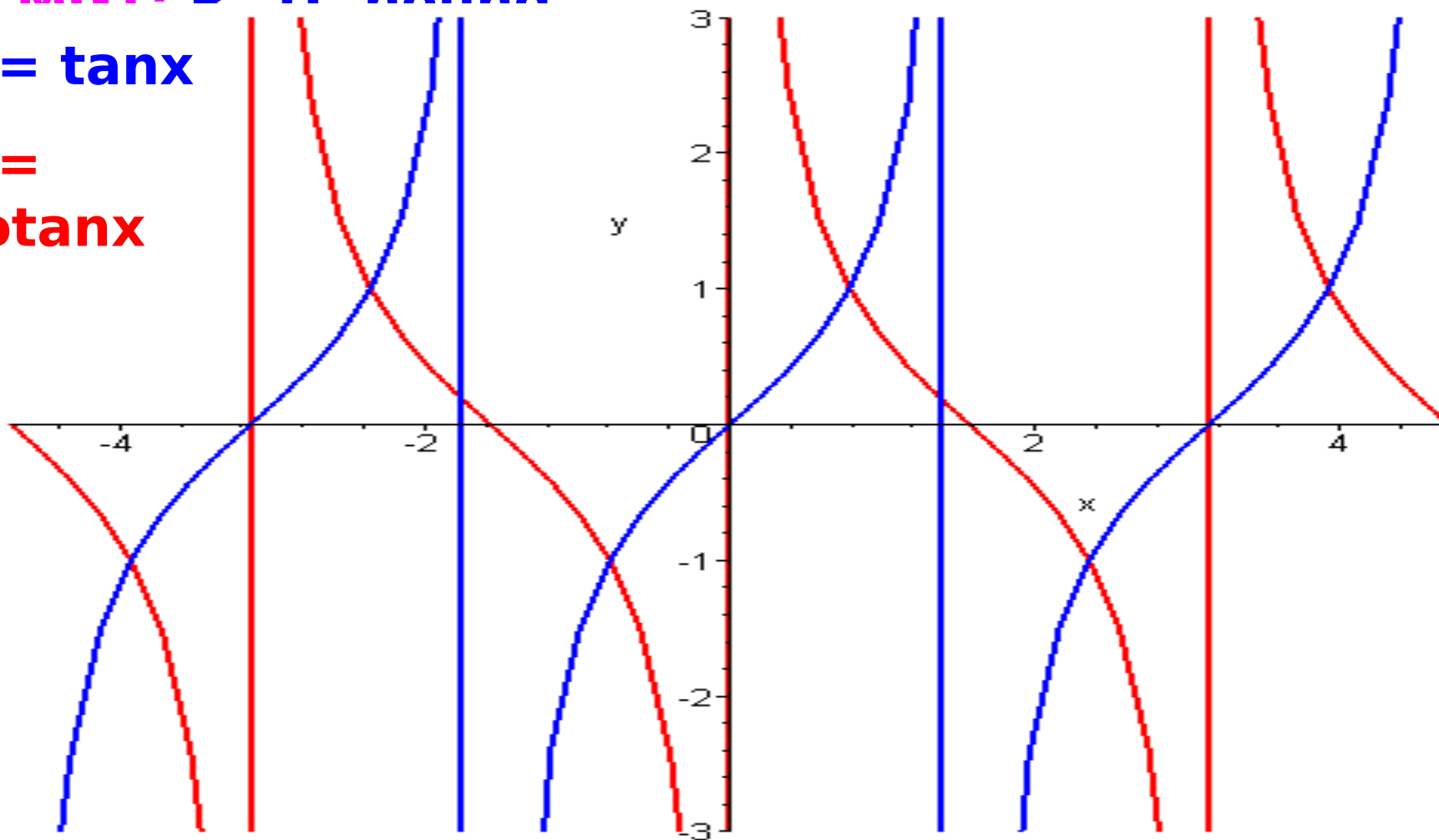
# HAØM LÖÖING GIAÙC: $\tan x$ , $\cotan x$

$$y = \tan x \quad (x \neq \pi/2 + k\pi), \quad y = \cotan x \quad (x \neq k\pi):$$

MCT. B TC hõing

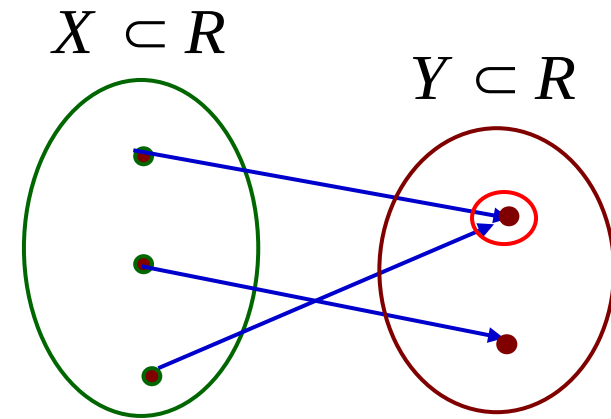
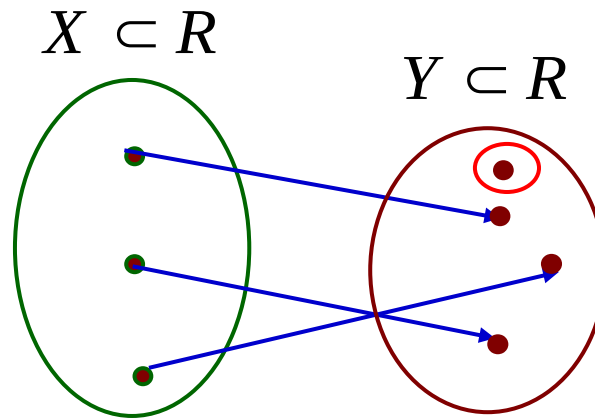
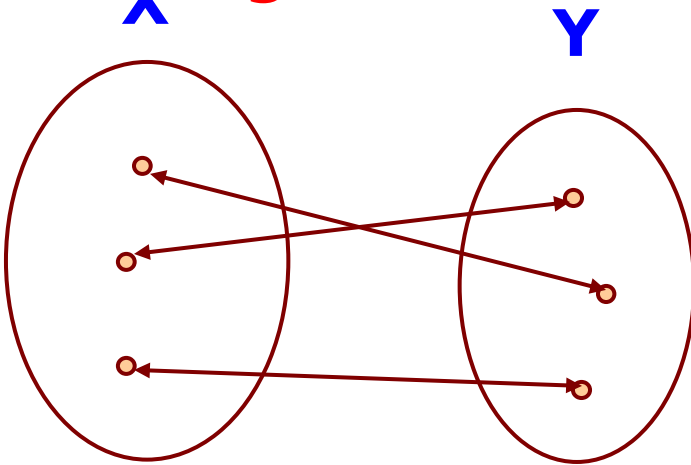
$$y = \tan x$$

$$y = \cotan x$$



# HAØM NGÖÖIC

Haøm soá  $y = f(x): X \rightarrow Y$  thoaû :  $\forall y \in Y, \exists! x \in X$   
sao cho  $y = f(x) \Leftrightarrow f$  laø moät song aùnh (töông  
öùng moät-moät)



• Khoâng laø s/a vì coù 1 gt y  
không coù x

• Khoâng laø s/a vì coù 1 gt y  
öùng vùi 2 gt x

$f$  song aùnh  $\Leftrightarrow$  vùi moii x, pt  $f(x) = y (*)$  coù

nghieäm x duy nhaát

**Ví  
dụ:**

• Hàm số  $y = f(x) = 2x + 3$  là song ánh trên  $\mathbb{R}$  vì  $f: \mathbb{R} \rightarrow \mathbb{R}$

và pt  $y = f(x) = 2x + 3$  có duy nhất

ngheäm  $x = (y - 3)/2$   
• Hàm số  $y = x^2$  ( $\mathbb{R} \rightarrow \mathbb{R}^+$ ) không là song ánh trên  $\mathbb{R}$  ( $x = \pm\sqrt{y}$ )

vì pt  $y = x^2$  không có duy nhất  
ngheäm

• Hàm số  $y = x^2$  là song ánh trên  $\mathbb{R}^+$  ( $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ ) ( $x = \sqrt{y}$ )

vì pt  $y = x^2$  không có duy nhất  
ngheäm

# HAØM NGÖÖIC

**Neáu**  $f : X \rightarrow Y$

$x \mapsto y =$

$f(x)$

laø song

ình

Ký hiệu haøm ngöôic :  $\varphi$

$= f^{-1}$

**thì**  $\varphi : Y \rightarrow X$

$y \mapsto x = \varphi(y)$ , vôi

$y = f(x)$

goïi laø haøm ngöôic của

$f$ .

**Cách tìm haøm ngöôic:**

1. Töö pt  $y = f(x)$ , giaûi tìm nghiệãm  $x = f^{-1}(y)$

2. Ñoái vai troø của  $x, y$  trong bieäu thöüc nghiệãm

của haøm  $y = f(x)$  vào  $y = f^{-1}(x)$  ñoái xöùng nhau  
g thaúng  $y = x$ .

## Ví dụ

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**1. Tìm hàm ngược của hàm số  $y = f(x) = 2x + 3$  trên  $\mathbb{R}$**

• **B1: giải pt  $y = f(x)$**

$$y = 2x + 3 \Leftrightarrow x = \frac{y - 3}{2}$$

**Biểu thức hàm ngược theo  $y$  :**

$$x = f^{-1}(y) = \frac{y - 3}{2}$$

• **B2: Nổi vai trò của  $x, y$  :**

$$y = f^{-1}(x) = \frac{x - 3}{2}$$

## 2. Tìm hàm ngược của hàm số $y = f(x) = x^2$ trên $\mathbb{R}^+$

$$\begin{cases} y = f(x) = x^2 \\ x \geq 0 \end{cases} \Leftrightarrow x = \sqrt{y} = f^{-1}(y)$$

**Vaä**  $y = f^{-1}(x) = \sqrt{x}$   
**y :**

## 3. Tìm hàm ngược của hàm số $y = f(x) = e^x$

**f :  $\mathbb{R} \rightarrow \mathbb{R}^+$ , vôi mỗi**  $y = f(x) = e^x \Leftrightarrow x = \ln y$

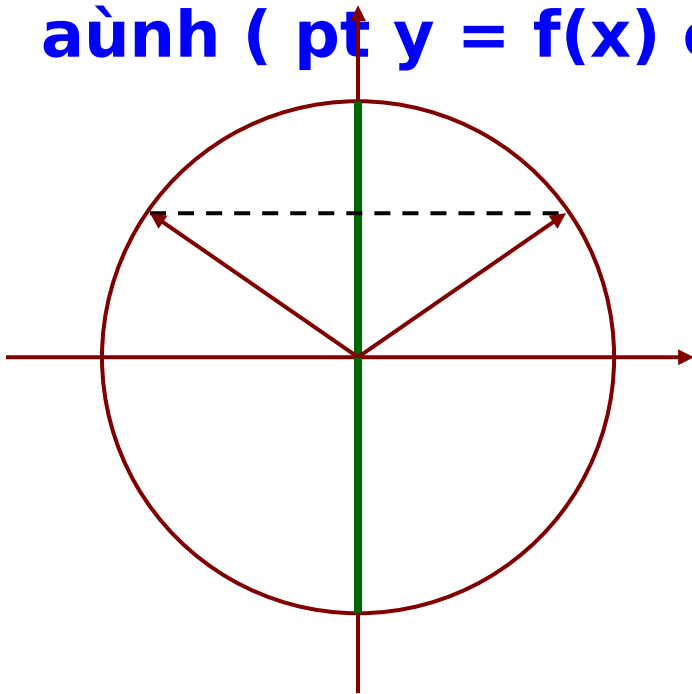
**y > 0 :**

**Vaäy**  $y = f^{-1}(x) = \ln x$   
**:**

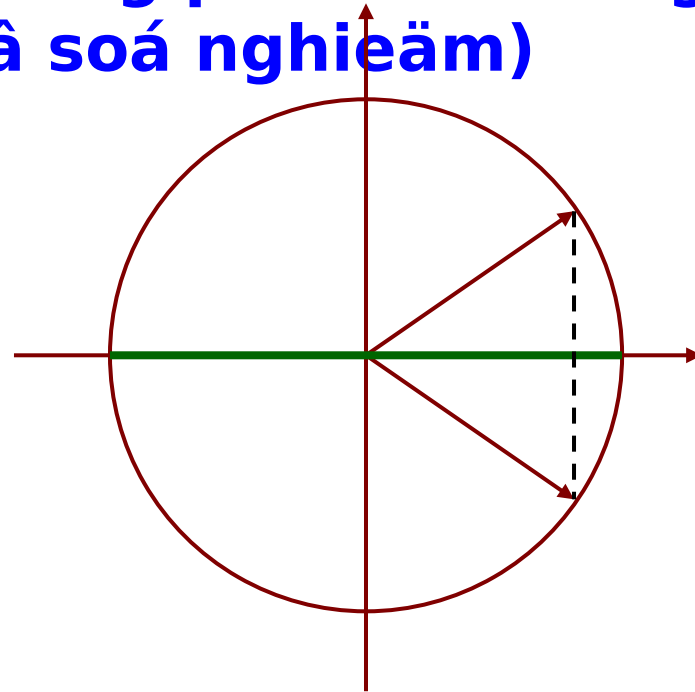
# HAØM LÖÖING GIAÙC NGÖÖIC

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• **Löu yù:** cauc haøm lööing giauc treân toaøn boã mieàn xauc ñòngh khoáng phaui laø song aùnh ( pñ  $y = f(x)$  còu voã soá nghiãm)

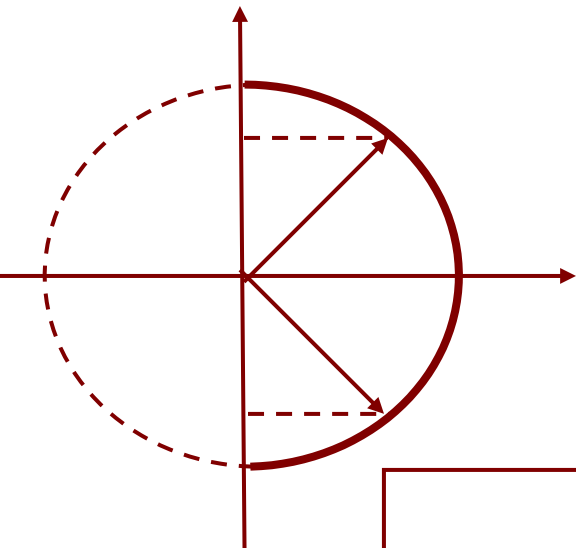


• Cauc goùc  $\varphi$  vaø  $\pi - \varphi$   
còu cøng giaù trò **SIN**



• Cauc goùc  $\varphi$  vaø  $\varphi$   
còu cøng giaù trò  
**COS**

# HAØM LÖÖING GIAÙC NGÖÖIC



$$y = \sin x \quad \text{laø song aùn h treân} \quad \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$
$$\sin : \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \xrightarrow{\text{s/a}} [-1, 1]$$

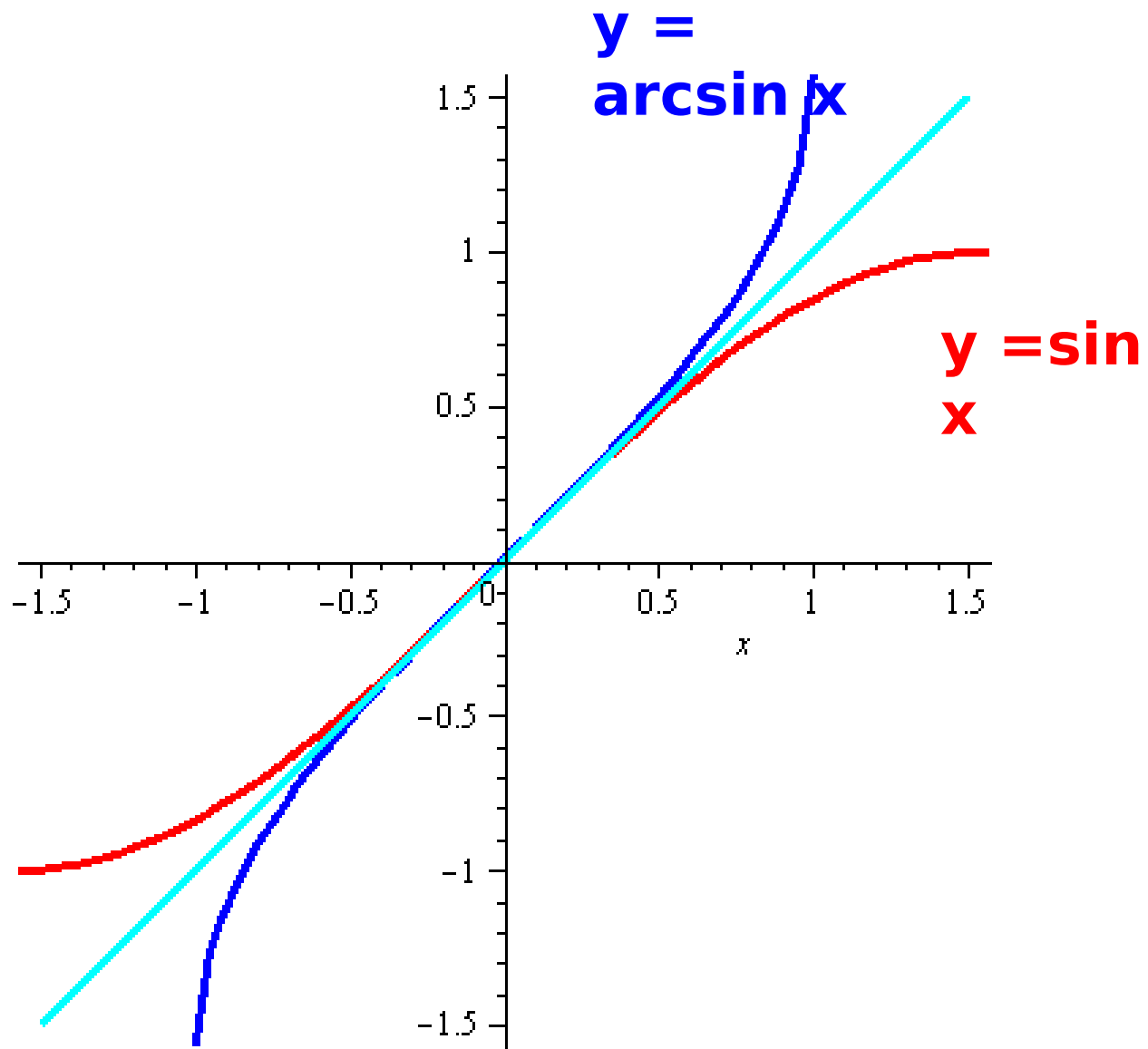
**toàn tãii haøm  
ngöôic**

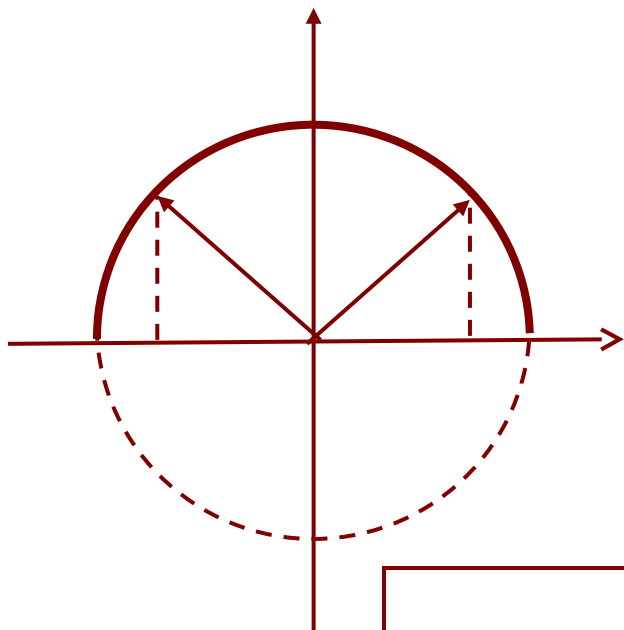
$$y = \sin^{-1} x = \arcsin x : [-1, 1] \longrightarrow \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

**Mieàn  
xaùc  
ñòn h**

**Mieàn  
giaù  
trò**

$$y = \sin x, x \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \Leftrightarrow x = \arcsin y$$





$y = \cos X$  **lặp song hành**  $[0, \pi]$   
**trên**

$$y = \cos X : [-1, 1] \longrightarrow [0, \pi]$$

**tồn tại hàm**  
**ngược**

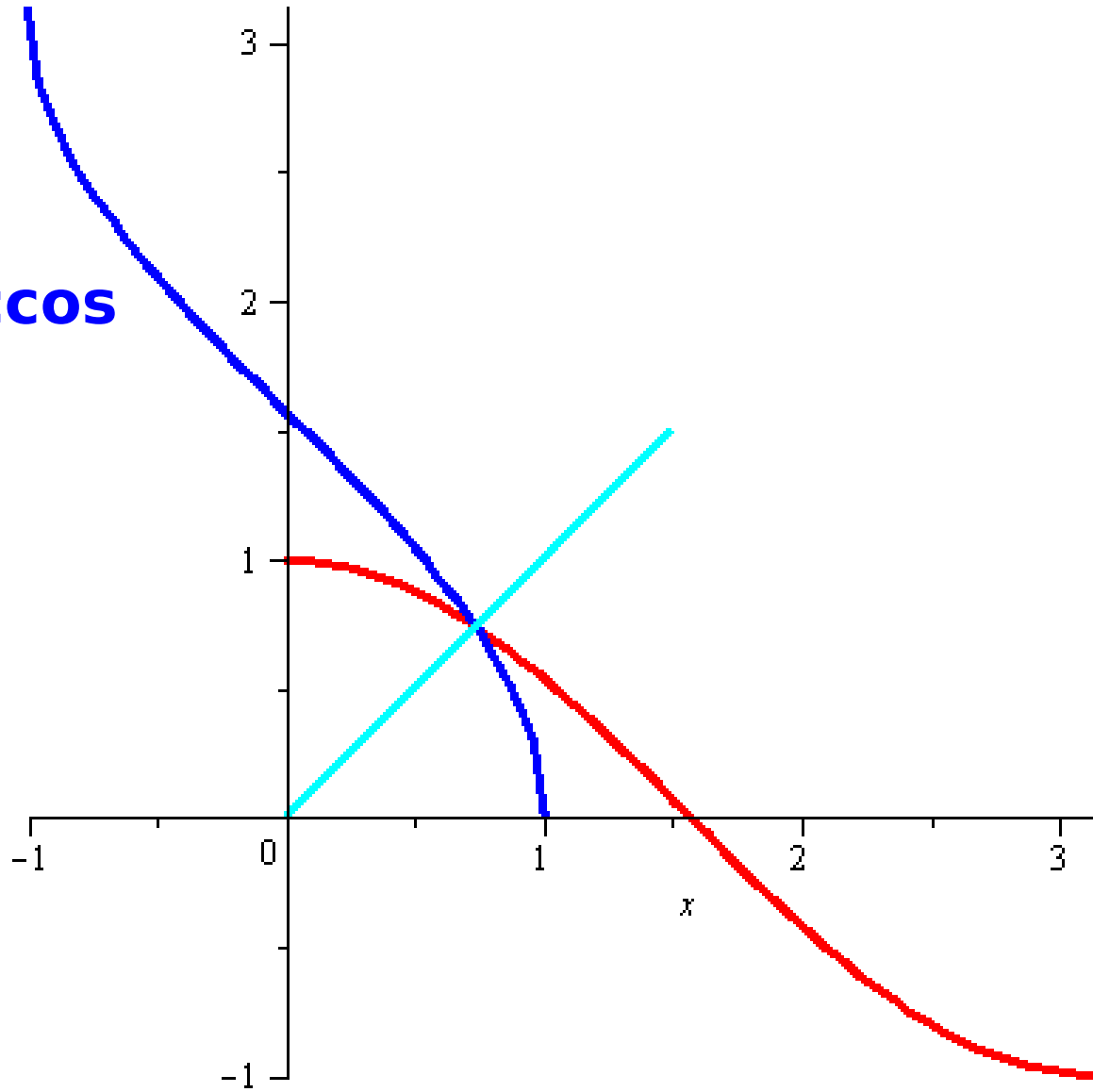
$$y = \cos^{-1} x = \arccos x : [-1, 1] \longrightarrow [0, \pi]$$

**Miền**  
**xaùc**  
**ñòngh**

**Miền**  
**giàu**  
**trò**

$$y = \cos X, X \in [0, \pi] \Leftrightarrow X = \arccos y$$

**$y = \arccos$   
 $x$**



**$y = \cos$   
 $x$**

## VÍ DUĚ

$$\sin 0 = 0 \Leftrightarrow \arcsin 0 = 0,$$

$$\cos 0 = 1 \Leftrightarrow \arccos 1 = 0,$$

$$\sin \frac{\pi}{2} = 1 \Leftrightarrow \arcsin 1 = \frac{\pi}{2},$$

$$\cos \pi = -1 \Leftrightarrow \arccos(-1) = \pi$$

$$\sin \frac{\pi}{6} = \frac{1}{2} \Leftrightarrow \arcsin \frac{1}{2} = \frac{\pi}{6},$$

$$\cos \left( \frac{\pi}{3} \right) = \frac{1}{2} \Leftrightarrow \arccos \frac{1}{2} = \frac{\pi}{3},$$

$$\sin \left( -\frac{\pi}{3} \right) = -\frac{\sqrt{3}}{2} \Leftrightarrow \arcsin \left( -\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3},$$

$$\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2} \Leftrightarrow \arcsin \left( -\frac{\sqrt{2}}{2} \right) = \frac{3\pi}{4}$$

$$y = \tan x : \text{song a\`nh} : \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \rightarrow R$$

$$\Leftrightarrow y = \arctan x : R \rightarrow \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

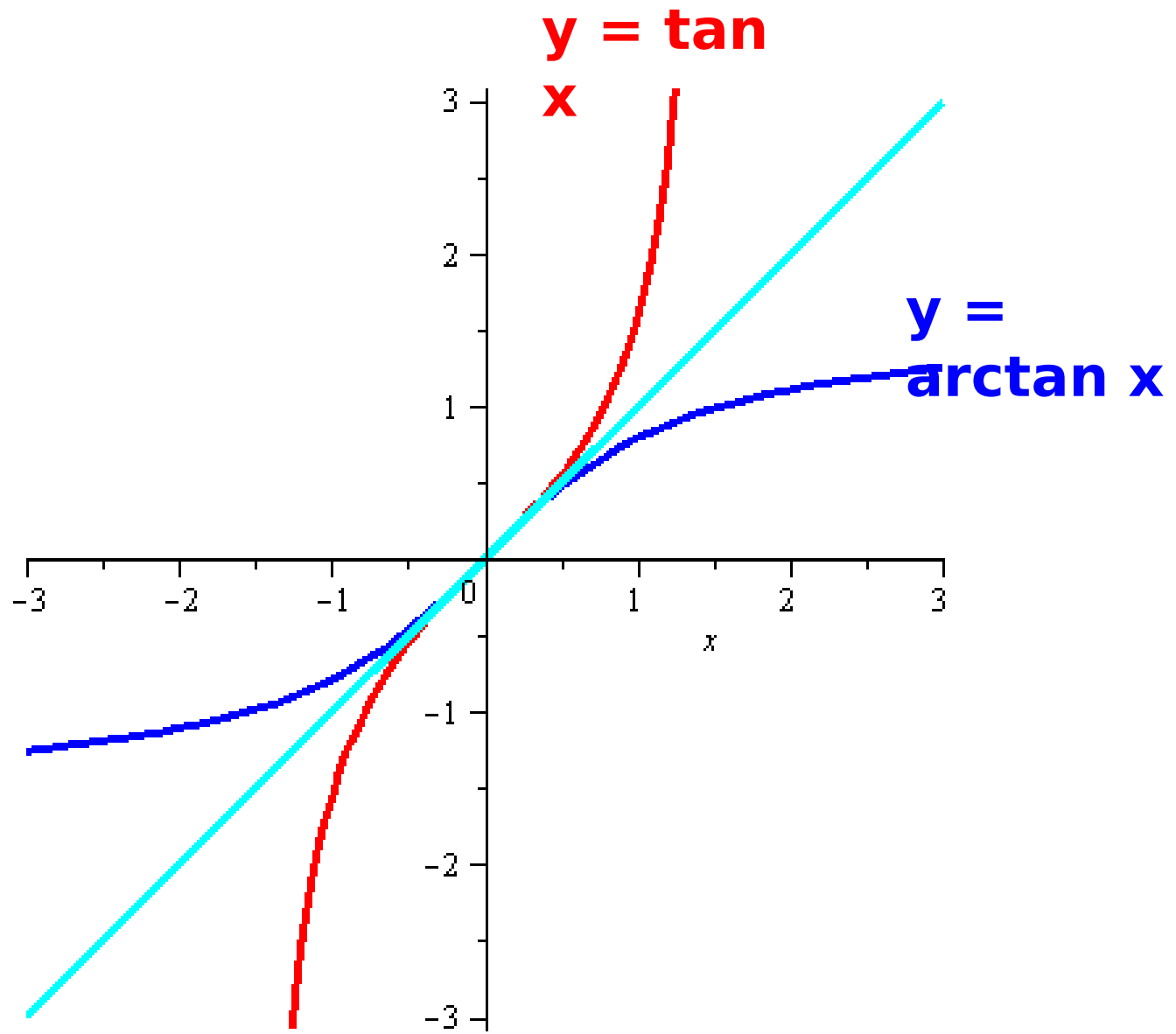
$$y = \cot x : \text{song a\`nh} : (0, \pi) \rightarrow R$$

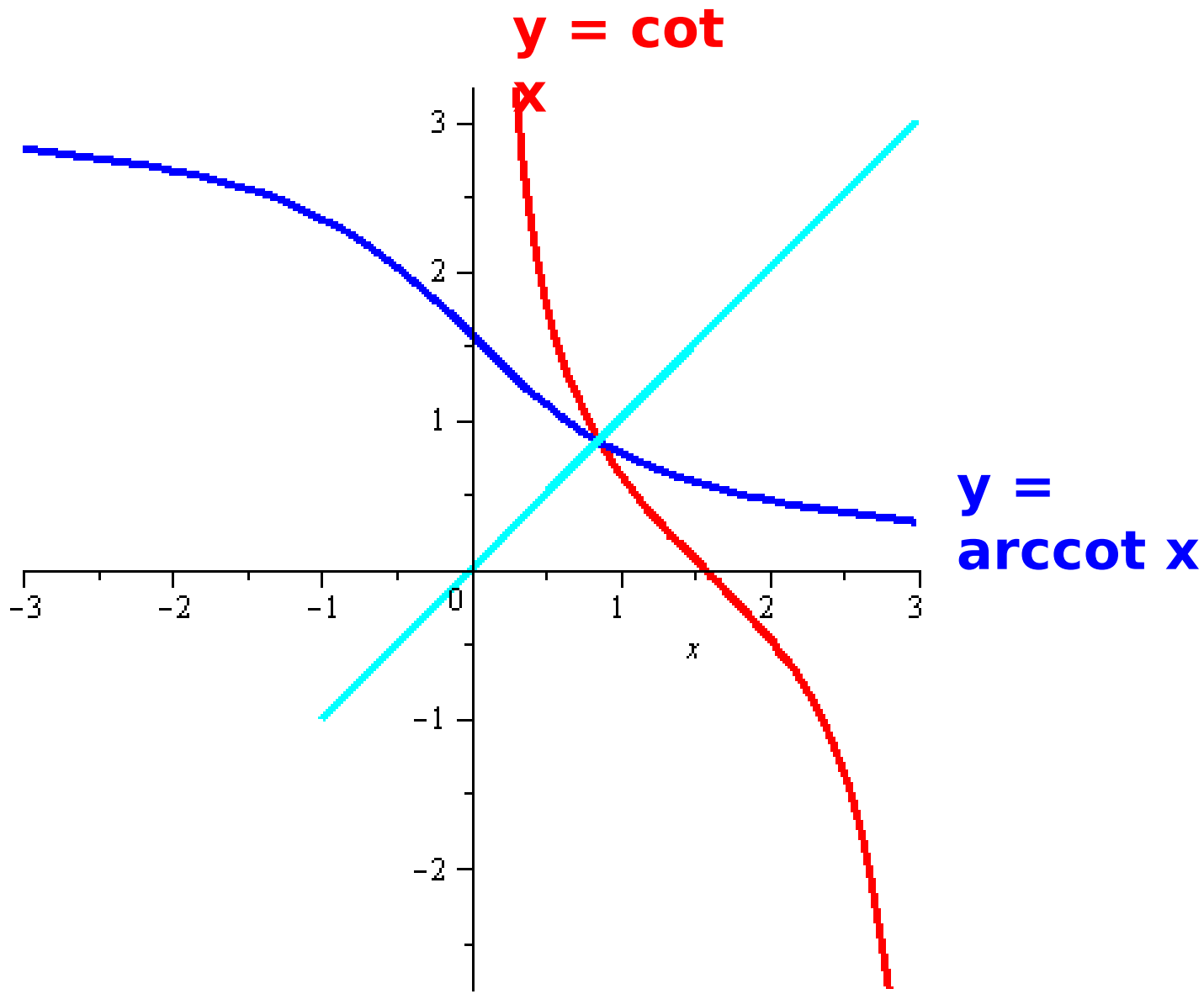
$$\Leftrightarrow y = \operatorname{arccot} x : R \rightarrow (0, \pi)$$

**Tính chất:**

$$\arcsin X + \arccos X = \frac{\pi}{2}$$

$$\arctan X + \operatorname{arccot} X = \frac{\pi}{2}$$





# HAØM HYPERBOLIC (Toaùn 1, ÑCK, trang 23 - 24)

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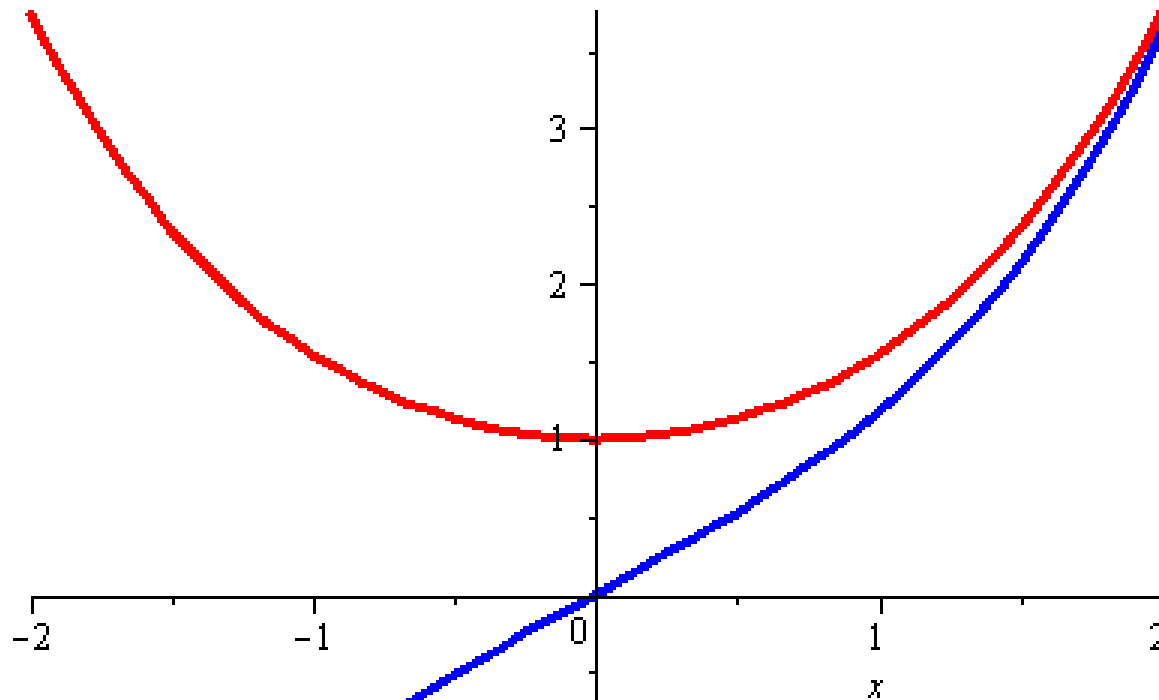
$$\sinh x = \operatorname{sh}x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \operatorname{ch}x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \operatorname{th}x = \frac{\sinh x}{\cosh x}, \quad \operatorname{coth} x = \operatorname{cth}x = \frac{\cosh x}{\sinh x}$$

- **Mieàn xaùc ñònh cuûa caùc haøm soá treân?**
- **Tính chaün leû?**

# ÑÒÀ THÒ HAØM Sinh x vaø Cosh x

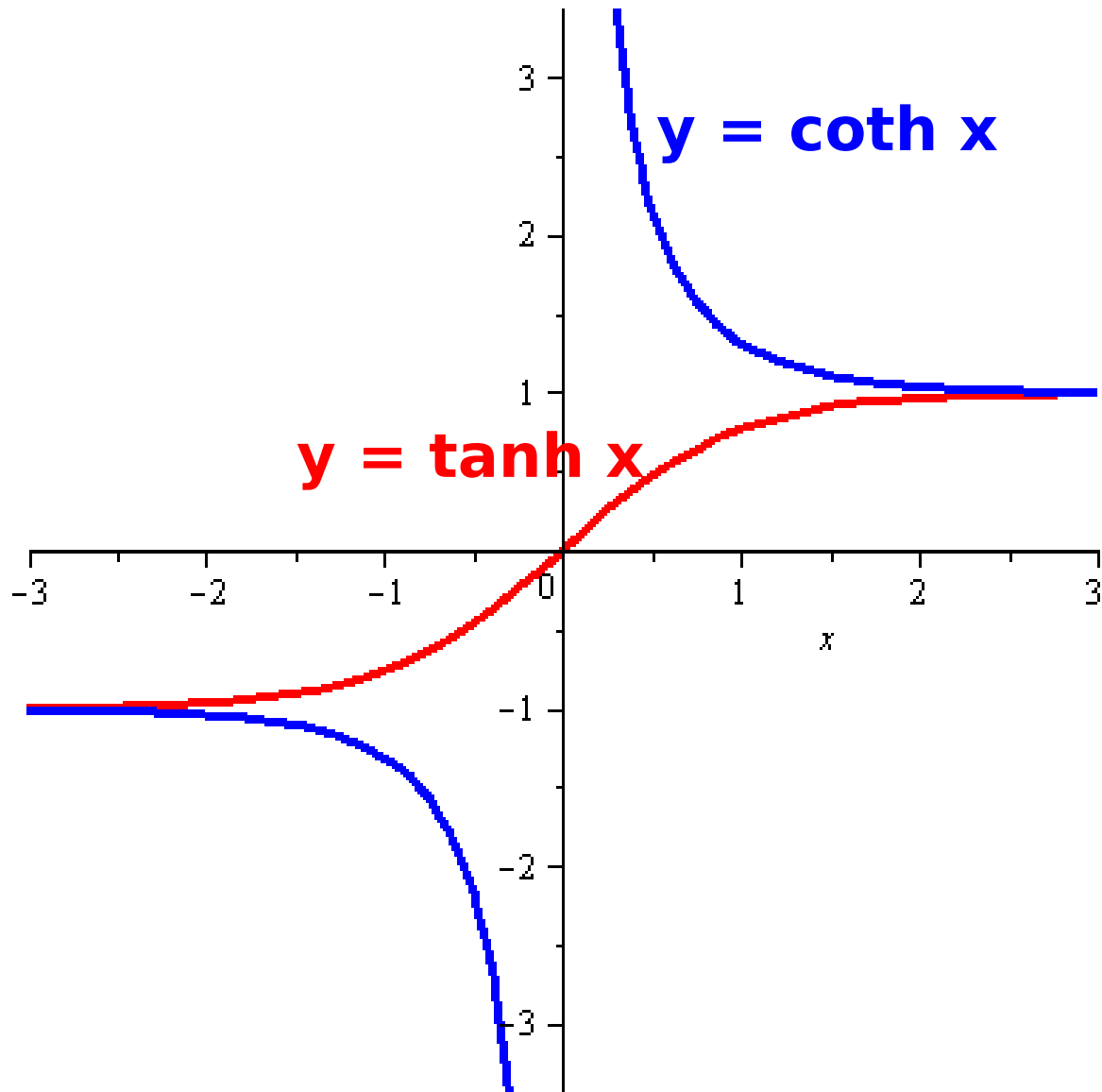
$$y = \cosh x$$



$$y = \sinh x$$

- $\cosh(x) \geq 1 \quad \forall x$
- $\sinh x < \cosh x \quad \forall x$

# ÑÒÀ THÒ HAØM $\tanh x$ vaø $\coth x$



**Ví 1/ Giải phương trình:**

**điều:**

$$\sinh(x) = 1$$

$$\Leftrightarrow e^x - e^{-x} = 2 \Leftrightarrow x = \ln(1 + \sqrt{2})$$

**2/ Chứng minh  $\cosh^2 x - \sinh^2 x = 1, \forall x$  (So sánh:**

**$\cos^2 x + \sin^2 x = 1$ )**

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= \left( \frac{e^x + e^{-x}}{2} \right)^2 - \left( \frac{e^x - e^{-x}}{2} \right)^2 \\ &= \frac{1}{2} + \frac{1}{2} = 1 \end{aligned}$$

# BAÛNG COÂNG THÖÙC THÖÙC HAØM HYPERBOLIC

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Coâng thöùc löôïng giaùc	Coâng thöùc Hyperbolic
$\sin^2 x + \cos^2 x = 1$	$\text{ch}^2 x - \text{sh}^2 x = 1$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\text{ch}(x \pm y) = \text{ch}x \text{ch}y \pm \text{sh}x \text{sh}y$
$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$	$\text{sh}(x \pm y) = \text{sh}x \text{ch}y \pm \text{sh}y \text{ch}x$
$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$	$\text{ch}(2x) = 2 \text{ch}^2 x - 1 = 1 + 2 \text{sh}^2 x$
$\sin(2x) = 2 \sin x \cos x$	$\text{sh}(2x) = 2 \text{sh}x \text{ch}x$
$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$	$\text{ch}x + \text{ch}y = 2 \text{ch} \frac{x+y}{2} \text{ch} \frac{x-y}{2}$
$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$	$\text{ch}x - \text{ch}y = 2 \text{sh} \frac{x+y}{2} \text{sh} \frac{x-y}{2}$