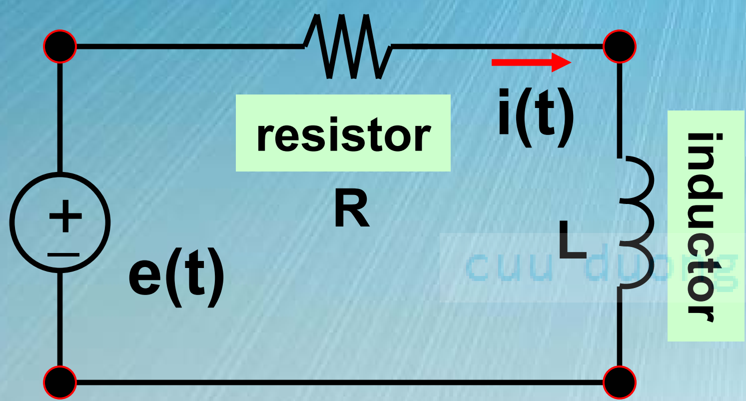
Chapter 6:

Applications of The Laplace Transform to Circuit Analysis

6.1: Laplace Circuit Solution:

- We wish to find the current:

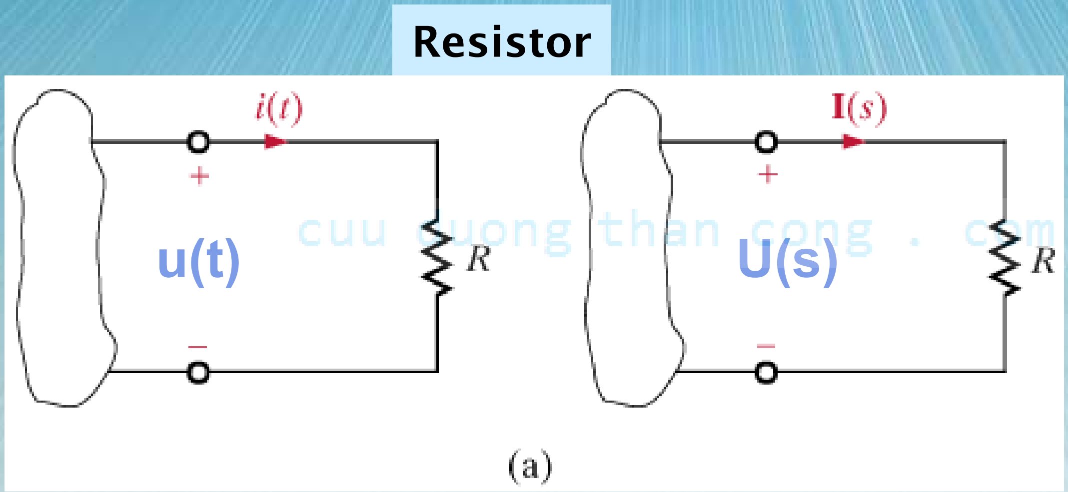


- Ohm law : $$R.i(t)+L\frac{di(t)}{dt}=e(t)$$

- Determine i(t) by using Laplace Transform.

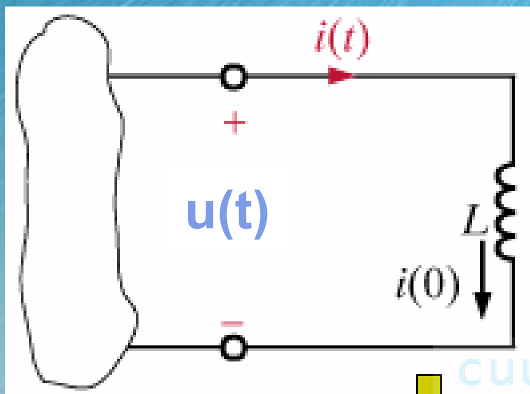
6.2: Circuit Element Models :

1) Resistor Model:



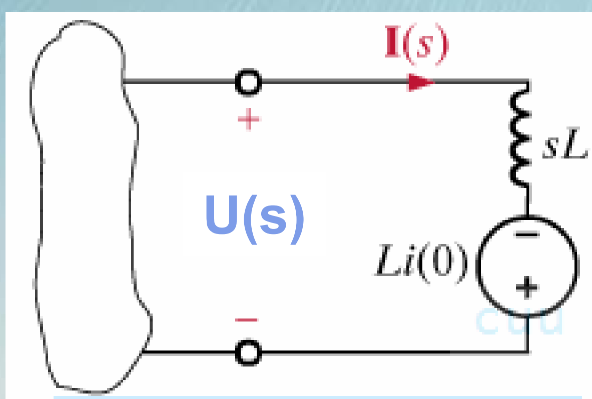
$$u(t) = Ri(t) \quad \Rightarrow \quad U(s) = RI(s)$$

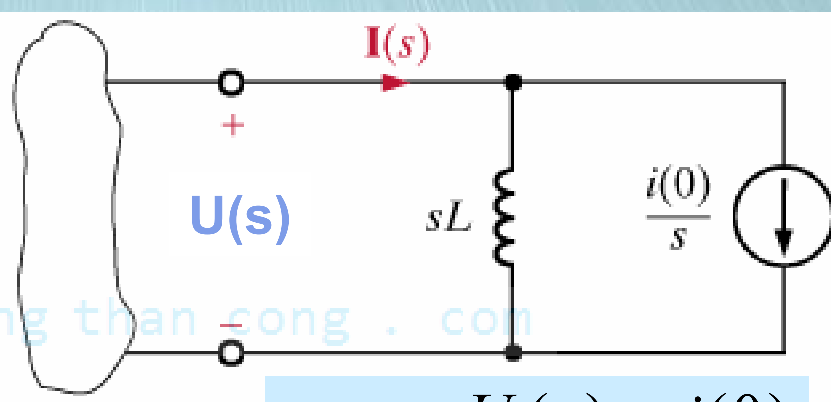
2) Inductor Models:



$$u(t) = L\frac{di}{dt}(t) \;\Rightarrow\; U(s) = L(sI(s) - i(0))$$

(sL= cảm kháng toán tử)

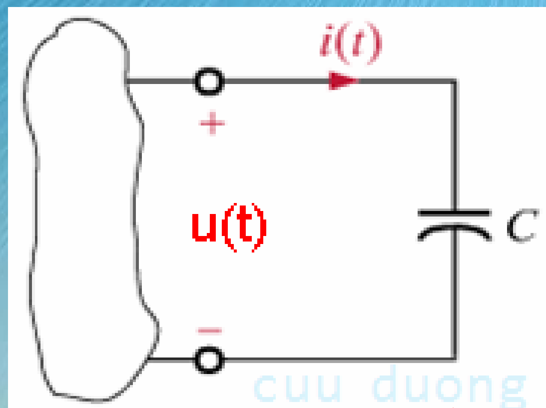




$$U(s) = sLI(s) - Li(0)$$

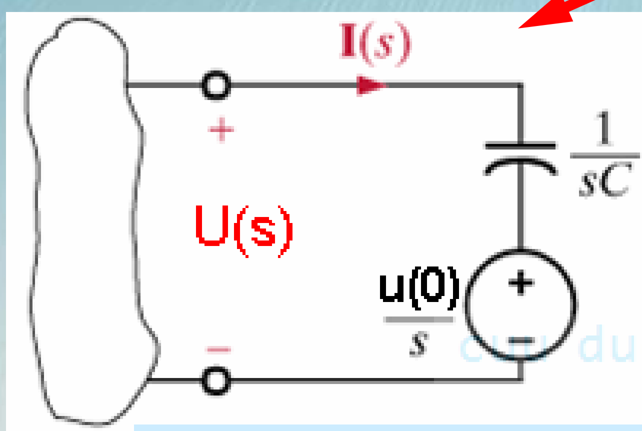
$$I(s) = \frac{U(s)}{sL} + \frac{i(0)}{s}$$

3) Capacitor Models:

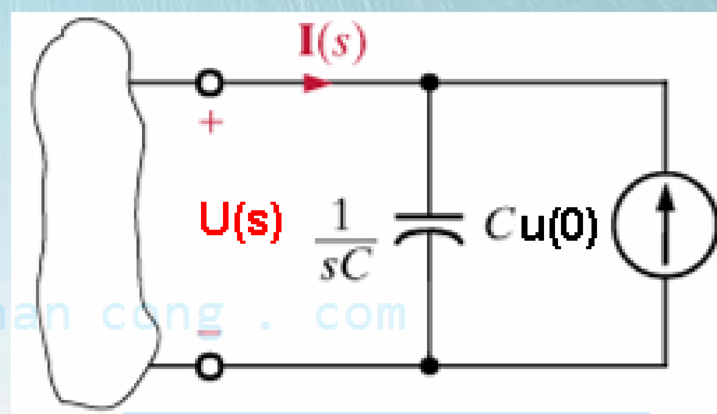


$$u(t) = \frac{1}{C}\int_0^t i(x)dx + u(0)$$

(1/sC= dung kháng toán tử)



$$U(s) = \frac{1}{sC} I(s) + \frac{u(0)}{s}$$



$$I(s) = sCU(s) - Cu(0)$$

4) Source Models:

a) Independent sources

$$e_S(t) \rightarrow E_S(s)$$

$$j_S(t) \rightarrow J_S(s)$$

b) Dependent sources

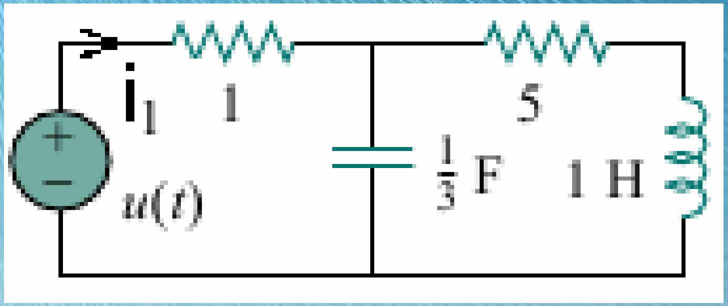
$$e_D(t) = k_4 i_C(t) \rightarrow E_D(s) = k_4 I_C(s)$$

$$j_D(t) = k_3 u_C(t) \rightarrow J_D(s) = k_3 U_C(s)$$

...

❖ Example1: The s-domain Circuit

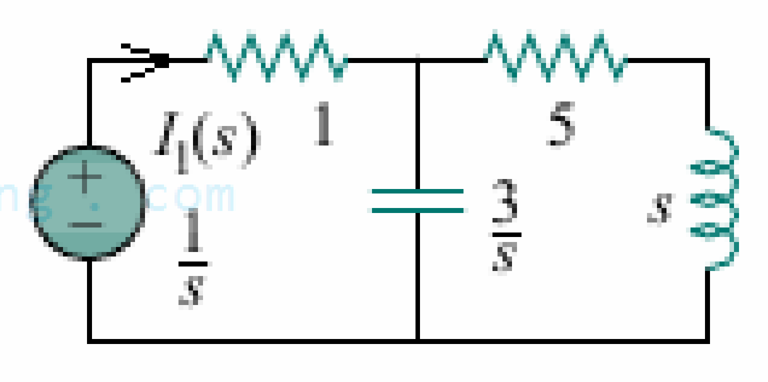
Transform the circuit to s domain , assuming zero initial conditions ?



$$u(t) \Rightarrow \frac{1}{s}$$

$$1H \Rightarrow sL = s$$

$$\frac{1}{3}F \Rightarrow \frac{1}{sC} = \frac{3}{s}$$

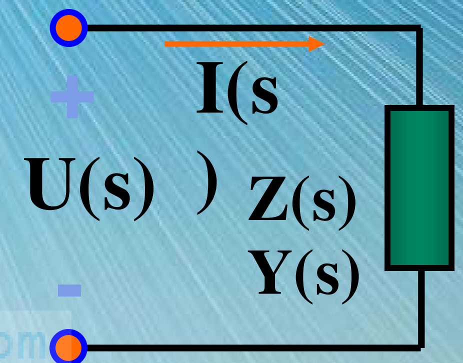


6.3: Transformed Circuit Laws :

1) Transformed Ohm Law:

$$U(s) = Z(s).I(s)$$

$$I(s) = Y(s).U(s)$$



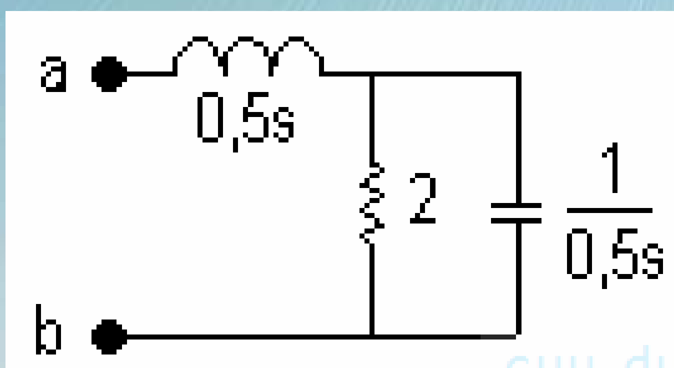
Với : $Y = \frac{1}{Z}$

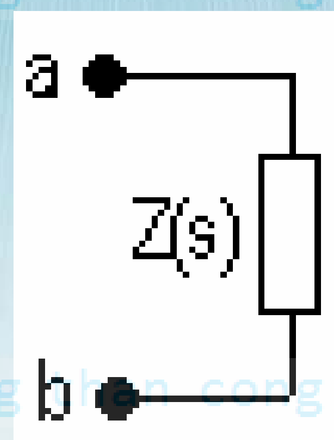
- **Z(s) : trở kháng , tổng trở toán tử (Ω)**
- **Y(s) : dẫn nạp , tổng dẫn toán tử (S) (℧)**

❖ Properties of Z(s) and Y(s) :

Z(s) và Y(s) đều tuân theo các phép biến đổi tương đương như điện trở và điện dẫn.

❖ **<u>Ví dụ :</u> Xác định trở kháng toán tử tương đương :**





$$Z = 0,5s + \frac{\frac{2}{0,5s}}{2 + \frac{1}{0,5s}}$$

$$\Rightarrow Z = 0,5s + \frac{2}{s+1}$$

2) Transformed KCL and KVL :

- **Luật KCL :** $$\sum_{node} \pm I_k(s) = 0$$

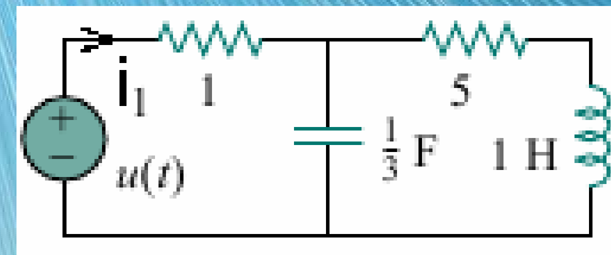
(Xét dấu như mạch điện trở)

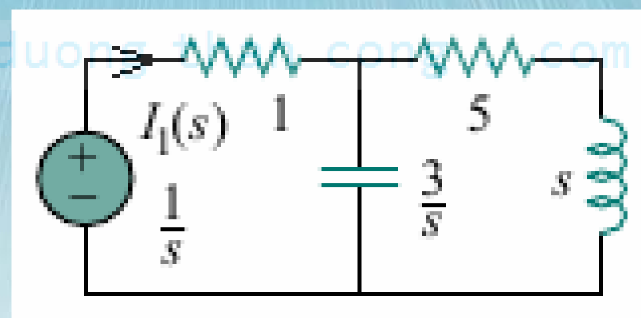
- **Luật KVL :** $$\sum_{loop} \pm U_k(s) = 0$$

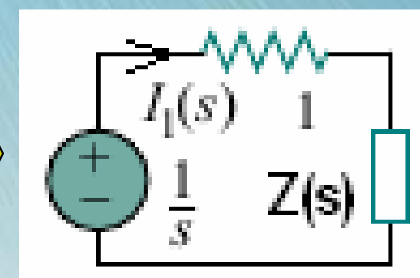
- **Do các luật Ohm và Kirchhoff viết cho mạch toán tử cũng tương tự viết cho mạch phức nên ta có thể áp dụng các phương pháp phân tích mạch điện trở đã học cho sơ đồ toán tử khi tìm ảnh Laplace bất kỳ.**

❖ Example1: s-domain Circuit Analysis

Find $i_1(t)$ in the circuit, assuming zero initial conditions ?







$$Z=\frac{\frac{3}{s}(5+s)}{\frac{3}{s}+5+s}$$

$$Z=\frac{3(s+5)}{s^2+5s+3}$$

$$\Rightarrow I_1=\frac{1/s}{1+Z(s)}=\frac{(s^2+5s+3)}{s[s^2+8s+18]}$$

$$\Rightarrow i_1(t)=\mathcal{L}^{-1}\{I_1(s)\}$$

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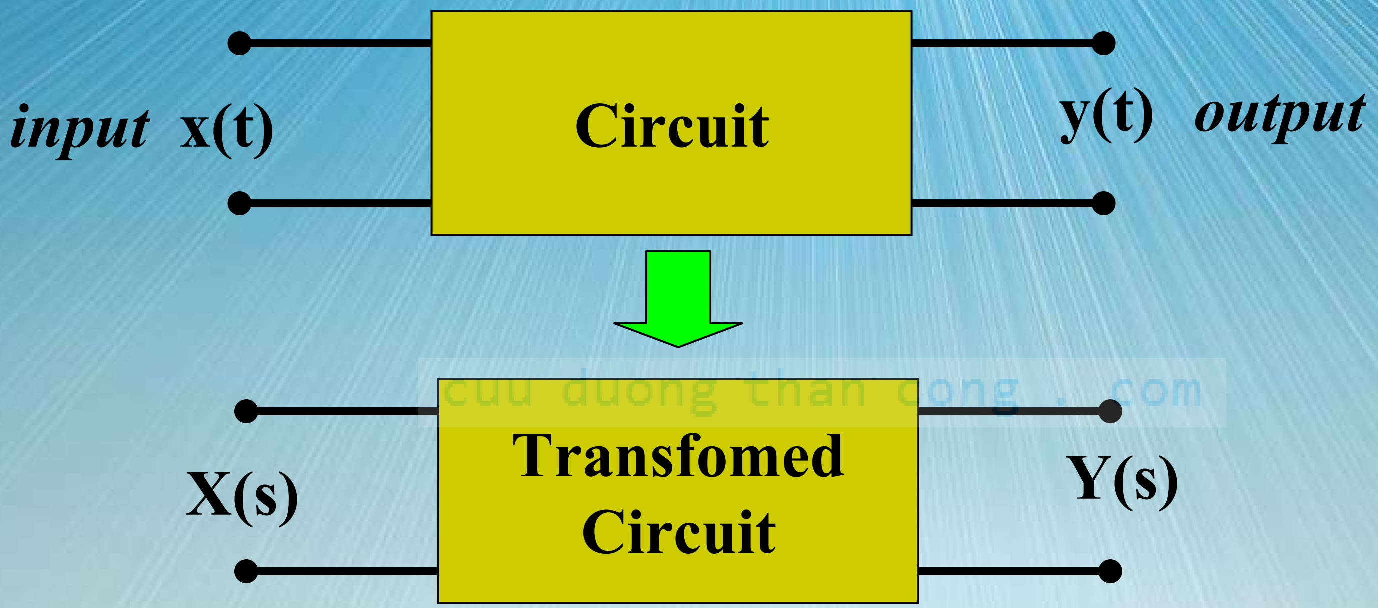
6.4: Transfer Function :



❖ H(s) is the ratio of the output Y(s) to the input X(s), assuming all initial conditions are zero.

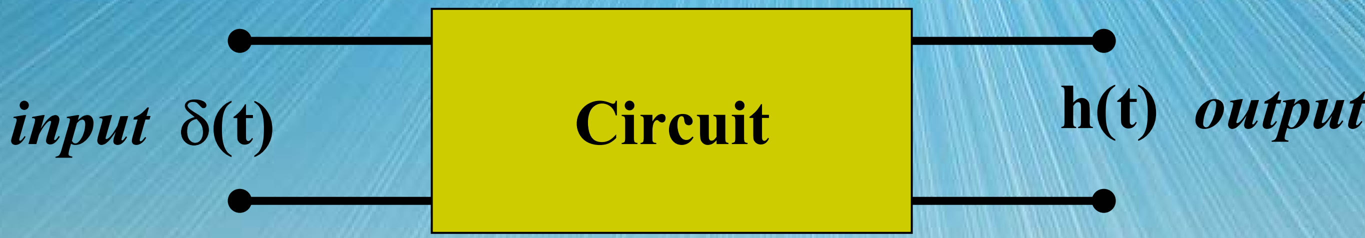
$$H(s) = \frac{Y(s)}{X(s)}$$

❖ Determine H(s) : in two ways

i. Using the circuit analysis methods .

ii. Using the propotional theorem to apply the ladder network.

❖ Unit Impulse Response h(t) :



$$\text{if } x(t) = \delta(t) \;\Rightarrow\; h(t) = \mathcal{L}^{-1}\{H(s)\} = \text{unit impulse response}$$

- **If we know h(t) → H(s) → Y(s) = H(s).X(s) and we can obtain the response y(t) of any input x(t) using y(t)=£$^{-1}${Y(s)} .**

- **If we know h(t): can obtain the response y(t) of any input x(t) using the convolution integral in time-domain (*See in the course named "Signal and System"*).**

❖Example1: Transfer Function

The output of a linear system is $y(t) = 10e^{-t}\cos 4t\,u(t)$ when the input is $x(t) = e^{-t}u(t)$. Find the transfer function of the system and its impulse response.

- **We have:** $X(s) = \frac{1}{s+1}$ **and** $Y(s) = \frac{10(s+1)}{(s+1)^2 + 4^2}$

- **And the transfer function :** $H(s) = \frac{10}{4}\frac{4}{(s+1)^2 + 4^2}$

- **The unit impulse function :** $h(t) = 2.5e^{-t}\sin 4t$